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MA722 - Foundations of Machine Learning Algorithms

Lab Problem Sheet - 3

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1. Write a python program that perform LU decompostion.
   - Define a square matrix A. Decompose A into:
     - A permutation matrix P
     - A lower triangular matrix L
     - An upper triangular matrix U
   - Print the matrices P, L, and U.
   - Verify that the decomposition satisfies the condition: $P * A = L * U$
2. Write a python program that perform LDU decompostion by:
   - Manual step-by-step Gaussian elimination method
   - Using scipy.linalg.lu
3. Write a Python program to perform Linear Regression on a given dataset.
   - Input: Two lists X and Y containing data points.
   - Implement Linear Regression in two ways:
     - From scratch using Gradient Descent.
     - Using the built-in Linear Regression class from scikit-learn.
   - Print the slope and intercept of the line.
   - Plot the data points and the regression line.
4. Consider a dataset with two predictor variables X_1 and X_2 and a response variable Y.
   (a) Implement Multiple Linear Regression from scratch by:
     - Construct the design matrix X (with a column of ones for the intercept).
     - Compute the regression coefficients (intercept, coefficient of X_1, coefficient of X_2).
     - Compute the predicted values $\hat{Y}$.

   (b) Calculate the **coefficient of determination** (R^2):
   $$R^2 = 1 - \frac{\sum(Y - \hat{Y})^2}{\sum(Y - \bar{Y})^2}$$
   (c) Verify the results using `scikit-learn`'s `LinearRegression` model and confirm that the intercept, coefficients, and R^2 match.

   (d) Plot the dataset points in **3D** and the fitted **regression plane**.
5. Write a Python program to compute the Moore–Penrose inverse (pseudoinverse) of a general m x n matrix.

- Define a matrix A of size m x n
- Compute the Moore–Penrose inverse A^+.
- Print both the original matrix and its pseudoinverse.
- Verify at least one of the conditions, for example: $AA^+A = A$

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